An Optimized Traffic Lights Scheduling

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***Abstract*—In this paper, we study on the optimal scheduling problem of the traffic lights on a map by utilizing the number of vehicles that use predefined routes. This paper considers modeling an optimization problem to minimize the number of vehicles on the map. We present the analytical solution of the optimization problem and the numerical solution approach by using an Open-Source library CVX implemented in Matlab. Also, we discuss the applications of our approach.**

***Index Terms*—constrained optimization, traffic light scheduling**

I. Introduction

Traffic congestion is one of the most important problems in urban transportation [1, 2]. Especially the cost of time and money that caused by congestion have significant effects on human lives. It was estimated that an average person loses over 100 hours in traffic jams every year. Also, the extra money spent on fuel and the value of time combined was more than $1000 per person in 2017 [3–5]. Besides, it can be argued that traffic jam has psychological effects on humans as well. Traffic jams may cause anger, high levels of stress, and behavioral anomalies for passengers as well as drivers [6–8].

Identifying the main causes of traffic congestion in urban areas is the primary step to relief the adverse effects of it. The structure of the roads is one of the most important indicators in traffic management. For example, the structure of the roads had to be designed by the boundaries of the buildings in central areas of old European cities. On the other hand, new types of transportation devices cannot be directly integrated into these conventional roads. Another significant indicator is human error. We may not be able to completely eradicate the human errors in traffic, however, we can supervise it by utilizing smart management approaches [3].

When we consider all these effects, we think that smart traffic scheduling solutions should be determined to ease the traffic conditions on roads. However, we need to define the traffic jam problem mathematically in order to solve it. We utilize the constrained optimization approach to find an optimal solution to traffic scheduling problem. However, we need to define some indicators to measure traffic congestion. There are several metrics have been articulated in [9, 10] such as travel time, speed, density, queue and distance. Nevertheless, these metrics also determine some of the constraints on the roads.

In this paper, we mainly focus on designing a constrained optimization problem by considering two types of road structures that can be used a basis to form more complex structures. We also discuss the analytical and numerical methods to reach an optimal solution of our problem. However, implementing an optimal solution on roads is as significant as finding a solution itself. Thus, we present some suggestions on applying our approach on urban roads.

This paper is organized as follows. In the following section, we provide a mathematical preliminary to form the problem of interest as a constrained optimization problem. In the next section, we introduce traffic flow structures that can be utilized in roads. In Section IV, we design a constrained optimization problem in accordance with the map type. In Section V, we present both the analytical and numerical solutions of the suggested optimization problem. In the last section, we conclude and discuss the main findings provided in this paper.

II. Mathematical Preliminary

Optimization is a process to achieve a feasible solution to any problem. To define an optimization problem, we must design the objective or cost function to be minimized or maximized. In the constrained optimization approach, we also examine the constraints of the system. We can define a constraint as a limitation that is dictated by the variables in the problem to prevent physical impossibilities. The general form of a constrained optimization problem can be presented as follows:

$$\min_{\mathbf{t}} \quad f(t_1, t_2, ..., t_n) \tag{1a}$$

$$\text{s.t.} \quad g_i(t_1, t_2, ..., t_n) = a_i, \ \forall \ i \in (1, 2, ..., p) \tag{1b}$$

$$h_i(t_1, t_2, ..., t_n) \leq b_j, \ \forall \ j \in (1, 2, ..., q). \tag{1c}$$

where the objective function to be minimized is given in (1a), the equality constraints are given in (1b), and the inequality constraints are given in (1c).

When we consider such constrained optimization problems, we may define the optimality conditions so-called Karush-Kuhn-Tucker (KKT) conditions. These conditions define a set of conditions to have an optimal solution for the problem. The first condition indicates that all constraints defined in the problem must be satisfied. The second condition guarantees that the cost will not be improved at the optimal solution. The third condition implies that the Lagrange multipliers and its corresponding constraint must be zero. The last condition mentions that the optimal Lagrange multipliers must be non-negative.

III. Traffic Flow Structures

The traffic flow can be managed in two different structures in roads: One-way traffic and two-way traffic [1]. One-way traffic allows vehicles to move in only one predefined direction on the particular roadway. However, it is more common that vehicles can travel in both directions at the same time on the same roadway. Therefore, we have two-way traffic structures which are divided into lanes. The importance of each traffic flow structure becomes evident if it is implemented on the correct roadway. On the other hand, a roadway can also be designed as a mixture of both structures if necessary.

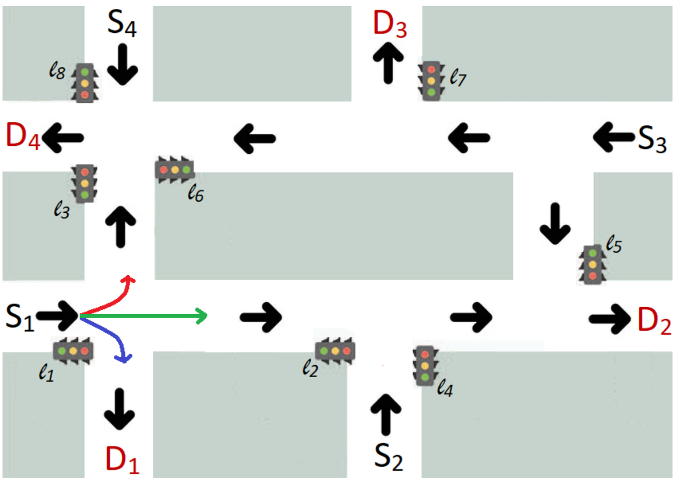

Fig. 1. A section of a city map with a one-way design.

Besides, we mainly consider strictly designated roadways to state our problem of interest. As can be seen from Fig. 1 the roadway is designed as one-way. The labels of S_i and D_j in Fig. 1 stand for possible Start and Destination points for a vehicle, respectively. For instance, a vehicle at S_1 can reach only D_1, D_2 and D_4 in accordance with the permitted lane directions. Also there are traffic lights distributed around the map in order to manage traffic flow. Their locations can be organized as the needs of the lanes.

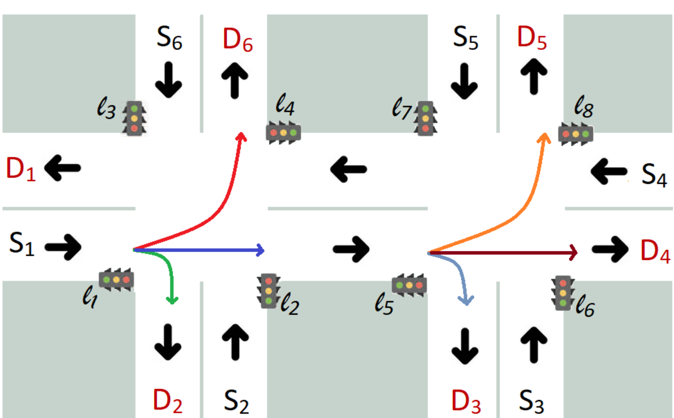

Fig. 2. A section of a city map with a two-way design.

The two-way structure can be more complicated than the one-way traffic flow designs since the junctions give more flexibility to vehicles for traveling to their destinations. In other words, a two-way structure gives more radius of action to vehicles. We illustrated a map of a two-way structured roadway in Figure 2. The labels of S_i and D_j in Fig. 2 stand for possible Start and Destination points for a vehicle respectively. We represented all the possible routes for a vehicle that starts from S_1. For example, a vehicle can travel from S_1 to D_2, D_3, D_4, D_5 and D_6 in the map illustrated in Fig. 2. The possible routes from another starting point to the destination can be determined in the same manner. We have again a number of traffic lights to control to optimize the number of vehicles in the two-way traffic. Although we can form several different roadway designs, the above two road structures can be considered as bases.

IV. Problem Definition

Most of the traffic light schedules are designated when they are first initialized. However, traffic light scheduling is essential to optimize the number of vehicles on roads. The management of the traffic light sequences by the vehicle density on roads allows us to minimize traffic congestion.

We utilize the constrained optimization technique to define our problem as follows:

$$\min \quad \sum_{i=1}^{n} \mathbf{r}_i - \mathbf{L}_c \mathbf{v} \tag{2a}$$

$$\text{s.t.} \quad \mathbf{v} - \mathbf{v}^{max} \leq 0 \tag{2b}$$

$$-\mathbf{v} \leq 0 \tag{2c}$$

$$\mathbf{L}\mathbf{v} - \sum_{i=1}^{n} \mathbf{r}_i \leq 0 \tag{2d}$$

$$\mathbf{v}^{min} - (\mathbf{r} - \mathbf{v}) \leq 0 \tag{2e}$$

where m defines the number of traffic lights, n defines the number of possible destination points on the map, $\mathbf{r}_i \in \mathbb{N}$ defines the number of vehicles at each route, $\mathbf{L}_c \in \mathbb{N}^{1\times m}$ is the choice of the situation of traffic lights, $\mathbf{v} \in \mathbb{N}^m$ holds the number of vehicles passes when green light on, $\mathbf{v}^{max} \in \mathbb{N}^m$ is the maximum vehicles can pass from each route i while l_j is green, $\mathbf{v}^{min}$ is the minimum vehicles should be waiting in queue at each route i while l_j is red.

We consider the cost function in (2a) to minimize the number of vehicles on the map in general. Since there are limited number of combinations for traffic lights to be green, the control center determines the L that minimizes the cost function most. The constraint (2b) determines the maximum number of vehicles that the road segment can hold. The constraint (2c) and (2d) impose the feasibility on the number of vehicles on the map. The last constraint indicates the minimum number of vehicles on each route.

V. Results

In this section, we first review the necessary conditions for optimal solutions by using the KKT conditions. In addition, we present a numerical example to show validity of the constrained optimization problem defined in Section IV.

A. Necessary Conditions for Optimal Solution

We first determine the necessary condition to find the optimal solution of the constrained optimization problem defined from (2a) to (2e). The Lagrangian can be presented as

$$\begin{aligned}\mathcal{L} = \sum_{1}^{n} r - L * v + \lambda_1(v - V_{max}) + \lambda_2(-v) \\ +\lambda_3\Big(L * v - \sum_{1}^{n} r\Big) + \lambda_4(V_{min} - r + v)\end{aligned} \tag{3}$$

The KKT conditions can be written as

$$\begin{aligned} -L - \lambda_1 - \lambda_2 + \lambda_3 L + \lambda_4 &= 0 \\ \lambda_1(v - V_{max}) &= 0 \\ \lambda_2(-v) &= 0 \\ \lambda_3\Big(L * v - \sum_{1}^{n} r\Big) &= 0 \\ \lambda_4(V_{min} - v) &= 0 \\ v - V_{max} &\leq 0 \\ -v &\leq 0 \\ \Big(L * v - \sum_{1}^{n} r\Big) &\leq 0 \\ V_{min} - v &\leq 0 \\ \lambda_1, \lambda_2, \lambda_3, \lambda_4, v &\geq 0. \end{aligned} \tag{4}$$

From the first condition in KKT conditions, we get $L(\lambda_3 - 1) - \lambda_1 - \lambda_2 + \lambda_4 = 0$. This leads to two cases:

$$\lambda_3 = 1 \text{ and } \lambda_4 - \lambda_1 - \lambda_2 = 0. \tag{5}$$

When we consider $\lambda_2 = 0$ from the third condition in (4), $\lambda_4 = \lambda_1$ is assured from (5). The complementarity conditions that are corresponding to λ_1 and λ_4 shows four cases:

Case 1: $v - V_{max} < 0$ leads to $v < V_{max}$ and $\lambda_1 = 0$. Thus $\lambda_4 = 0$. Then we conclude that $V_{min} + V_{max} < r$. This result can lead one of the optimal solutions.

Case 2: $v - V_{max} = 0$ leads to $v = V_{max}$. However, we may have two sub-cases for the lagrange multiplier λ_1 such as $\lambda_1 = 0$ and $\lambda_1 > 0$. We have considered the former one in above case. We now consider $\lambda_1 > 0$. Thus $\lambda_4 > 0$. Then we get $V_{min} + V_{max} = r$ which is valid and can lead one of the optimal solutions.

Case 3: $V_{min} - r + v < 0$ leads directly the same conclusion in case 1.

Case 4: $V_{min} - r + v = 0$ leads directly the same conclusion in case 2.

Above conditions indicates the necessary conditions to have optimal solutions of our problem of interest.

B. A Numerical Example

On the other hand, we present a numerical example by considering the map illustrated in Fig. 1. We have utilized an Open-Source library CVX and Optimization Toolbox in Matlab to solve our constrained optimization problem given from (2a) to (2e).

There are eight routes defined in Fig. 1. The number of vehicles on each route given as

$$\mathbf{r} = [30,\ 45,\ 85,\ 33,\ 27,\ 92,\ 25,\ 49]^T.$$

Therefore, there are currently 386 vehicles on road in total. The maximum and minimum capacities for each route are given as

$$\begin{aligned}\mathbf{v}_{max} &= [0,\ 50,\ 50,\ 50,\ 50,\ 50,\ 0,\ 50]^T, \\ \mathbf{v}_{min} &= [0,\ 15,\ 15,\ 15,\ 15,\ 15,\ 0,\ 15]^T.\end{aligned}$$

The choice of the situation of traffic lights $\mathbf{L}_c$ can be chosen as one of the rows determined in the following matrix,

$$\mathbf{L} = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Above matrix shows that there are nine combinations for eight traffic lights in this map. The value 1 in matrix $\mathbf{L}$ indicates the green light while 0 is the red light.

The choice of $\mathbf{L}_c$ changes the resultant cost in problem defined (2) immediately. Therefore, we need to assess all the possible traffic light configuration defined in $\mathbf{L}$ to find the optimal solution for $\mathbf{v}$. The following table represents the optimal $\mathbf{v}$ for each traffic light configuration and the cost in accordance with the optimal $\mathbf{v}$.

TABLE I

Choice of L (row #)	optimal $\mathbf{v}$	cost
1	**$[0, 30, 50, 0, 0, 0, 0, 0]^T$**	**306**
2	$[0, 0, 50, 18, 0, 0, 0, 0]^T$	318
3	$[0, 0, 50, 0, 12,0 , 0, 0]^T$	324
4	**$[0, 30, 0, 0, 0, 50, 0, 0]^T$**	**306**
5	$[0, 0, 0, 18, 0, 50, 0, 0]^T$	318
6	$[0, 0, 0, 0, 12, 50, 0, 0]^T$	324
7	$[0, 30, 0, 0, 0, 0, 0, 34]^T$	322
8	$[0, 0, 0, 18, 0, 0, 0, 34]^T$	334
9	$[0, 0, 0, 0, 12, 0, 0, 34]^T$	340

Above results represent that there are two different choices of traffic light configuration makes the cost minimum in map illustrated in Fig. 1. Therefore, either of the possible green-light sequence can be chosen for minimizing the number of vehicles on the road.

The following matrix $\mathbf{L}$ is formed in accordance with the possible traffic light configurations for the map given in Fig. 2.

$$\mathbf{L}_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

There are sixteen possible actions for the traffic lights to turn green at the same time. We can directly implement one choice of configuration as $\mathbf{L}_c$ in our problem definition. Then we can find an optimal solution by taking into account the current road conditions for map in Fig. 2.

VI. Conclusion

In this paper, we study a constrained optimization technique to minimize the traffic congestion on the roads. We review two types of traffic flow structures designed in one-way and two-way fashions. These two structures can be utilized to form the urban roadways. Therefore, roads may be designed to be more complex and intertwined which can cause traffic congestion. We propose a mathematical problem definition to minimize the number of vehicles on each route, regardless of the road design. We mainly try to supervise the traffic light configurations on routes. We have defined the necessary conditions to have the optimal solution of our problem. We have also provided a numerical example to validate our findings.

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